

A common and concise formats approach to a Key-Curve construction for generating crack extension data in C(T) specimens

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ABSTRACT: The construction of a J - R curve for a ductile material following Standard ASTM E 1820-15 (2015) requires information on the stable crack extension process. According to E1820, the resistance curve may be obtained from a single specimen test, in which the crack size is measured simultaneously with force and displacement by the unloading compliance, potential drop, or normalization procedures. Based upon the Common and Concise Formats, Donoso and Landes developed the “*crack growth law*” and the “*intercept method*” as an alternative to obtain crack sizes in a test that shows stable crack extension, but have only force-displacement data and the initial and final crack sizes available. These alternative methods are now supplemented by an improved key-curve construction based on these formats, and are applied to C(T) test data in which the crack extension values are insufficient to produce a valid J - R curve.

KEYWORDS: Common format; Crack growth law; Fracture toughness; J-R Curve; Stable crack extension

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RESUMEN: *Un enfoque de los Formatos Común y Conciso para la construcción de una curva maestra para generar valores de extensión de grieta en probetas C(T).* La construcción de curvas J - R para un material dúctil, acorde con la Norma ASTM E1820-15 (2015), requiere de información del proceso de extensión estable de grieta. Según la Norma, la curva de resistencia puede ser obtenida de un ensayo con solo una probeta, en el cual el tamaño de grieta es medido simultáneamente con fuerza y desplazamiento por los métodos de cambios de flexibilidad, caída de potencial, y normalización. Basándose en los Formatos Común y Conciso, Donoso y Landes desarrollaron la “*ley de crecimiento de grieta*” y el “*método del intercepto*” como alternativas para obtener tamaños de grieta de un ensayo que exhibe extensión estable de grieta, pero del cual se dispone solo de datos fuerza-desplazamiento, y tamaños de grieta inicial y final. Ambos métodos alternativos se ven complementados por la construcción de una curva maestra, basada en estos Formatos, aplicada a probetas C(T) en las cuales los valores experimentales de extensión de grieta son insuficientes para producir una curva J - R válida.

PALABRAS CLAVE: Curva J-R; Extensión estable de grieta; Formato común; Ley de crecimiento de grietas; Tenacidad a la fractura

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1. INTRODUCTION

The evaluation of the fracture toughness of a material, as a measure of its resistance to crack extension, has been a relevant issue for the past five decades. Both in linear elastic as well as in elastic-plastic fracture, the interest in producing a value for a fracture-related property has prompted an enormous body of research. Recently, Zhu and Joyce (2012) published a detailed technical review of fracture toughness testing, evaluation and standardization for metallic materials that show either ductile or brittle behavior. While a brittle material shows rapid and unstable crack extension, a ductile material is characterized by slow and stable crack growth during the test. In the latter case, the fracture toughness is often evaluated with ASTM E1820-15 (2015), both for the construction of the resistance, J - R curve, and for the evaluation of the initiation toughness J_{Ic} .

The construction of J - R or J - Δa curves for a ductile material per E1820 requires knowledge of the stable crack extension process. Thus, the J - R curve may be obtained from a single specimen test, by measuring crack length, a , concurrently with force P , and load-line displacement v . Evaluation of the actual crack size may be achieved by elastic unloading and reloading compliance changes, or by other similar techniques. Thus, knowledge of the actual crack extension value, $\Delta a = a_i - a_o$, where a_i and a_o , are current and initial crack size, respectively, is essential in J - R testing. As part of the construction of the resistance curve, E1820 requires a minimum number of J - Δa points between the 0.15 and 1.5 mm exclusion lines. Once this requirement has been met, a provisional value of the initiation fracture toughness, J_Q , may be obtained as the intercept of the J - Δa curve with the 0.2 mm offset line given by $J = 2\sigma_y(\Delta a - 0.2)$, where σ_y is the average of the yield and the ultimate tensile stress of the material. This provisional value, J_Q , must then be validated as a “critical value” J_{Ic} under E1820 requirements.

Many of the classical methods for developing the J - R curve do not work under some testing conditions and a normalization method like the key curve methodology is a solution. Several attempts have been made to construct the J - R curve, to calculate the single value J_{Ic} , or to produce a value that represents toughness, when there are an insufficient number of crack extension points, or the crack size data is missing. Either at elevated temperatures, or when the test is carried out at some hostile environment, it may be extremely difficult to measure the change in crack size as the test is carried out. Joyce *et al.* (2001) proposed a method to be applied to dynamic loading, in which crack extension measurements are not viable, in order to make it possible to construct a dynamic J - R curve. On the other hand, Pehrson

and Landes (2006) integrated the load vs load-line displacement curve to the point of maximum load, obtaining good J_Q estimations for $W = 50$ mm specimens. For the materials analyzed, they pointed out that J_Q may be estimated accurately by integrating to 99% of maximum load, without the need to construct a J - R curve.

In this context, Donoso and Landes (1994) and Donoso and Landes, (2001) developed methodologies that relate all three variables of a fracture toughness test: force P , displacement v , and crack size a . In the elastic-plastic regime, the method is referred to as the “Common Format”, whereas the “Concise Format” accounts for the stress intensity factor K and specimen compliance in elastic behavior. Although the Common and Concise (C&C) Formats were initially formulated for the analysis of elastic-plastic behavior of blunt-notch C(T) specimens of unit size ($W = 50$; $B = 25$ mm), they rapidly evolved to allow evaluation of crack extension of pre-cracked test specimens. Thus, based upon the C&C formats, Donoso, Zahr and Landes (DZL) proposed the notion of a “*crack growth law*” (Donoso *et al.*, 2005a; Donoso *et al.*, 2005b) as the core of a method for obtaining the actual crack size in a fracture test of a C(T) specimen showing stable crack extension. Under limiting experimental conditions, with only the force-displacement, P - v , record and initial and final crack sizes, a_o and a_f , available, this method is an alternative to the unloading compliance, potential drop, and normalization procedures of ASTM E1820-15 (2015). Further improvements based on the DZL crack growth law gave rise to the “*intercept method*” by Donoso *et al.* (2008) and Donoso *et al.* (2009) as another way of generating the amount of stable crack growth, when there is scarce information — or none at all — on the values of the actual crack size as a function of the P - v data.

As an addition to the methodologies discussed above, this work will show how to construct a “key curve” based on the Common and Concise (C&C) formats to help in the process of generating crack extension data. The method rests upon the construction of the P - v curve in a test in which the specimen crack size remains constant, at a value $a = a_o$; in other words, the result of testing a blunt-notch specimen. In these circumstances, there would be ideally no crack extension - only strain hardening of the specimen material - and the force-displacement curve would rise continuously, showing no maximum except for the final value of the force at the termination of the test. Since it is not simple to find information for pre-cracked C(T) specimens together with data collected from identical blunt notch specimens, the latter will be generated presently with the C&C Formats.

The key curve concept has been used, among others, by Joyce (1983), Andrews (1985) and

Candra *et al.* (2002) to circumvent the difficulties imposed by testing under difficult conditions: dynamic testing, temperatures different from room temperature, or testing in a medium in which using a displacement gage is not feasible. The common feature in these works is the use of both components of the displacement separately, the elastic and the plastic displacements. In the first case, the elastic compliance is capital in relating the progress of the test to the crack size achieved. In the present work, this is accomplished by using the C&C Formats, and represents an improvement of the work by Donoso *et al.* (2009). This will be shown presently.

2. MATERIALS AND METHODS

2.1. Fundamentals of the C&C Key Curve construction

Figure 1 shows the experimental force-displacement, P - v , curve for a vintage A508 C(T) specimen of dimensions width $W = 50.4$ mm and thickness $B = 25.2$ mm, for which the crack size a is known at each P - v pair (Donoso *et al.*, 2009). The final point shown in Fig. 1 has the values $v = 2.84$ mm, $P = 37.94$ kN, and $a = 31.62$ mm. The continuous curve, or “key curve” (KC) is constructed with the C&C Formats as a blunt notch P - v curve with constant ligament $b_o = W - a_o$, where a_o is the initial crack size. For this specimen, $a_o = 26.19$ mm.

The key curve is constructed by adding the elastic and the plastic displacement at any given value of force, both displacements being dependent on crack (ligament) size. Since both a blunt-notch specimen as well as one undergoing stable crack extension usually display a plastic displacement larger than the elastic component, the generation of the key curve

is carried out with the Common Format Equation, CFE (Donoso and Landes, 1994), which relates force P , plastic displacement v_{pl} , and ligament size b in a C(T) fracture specimen, as given by Eq. (1):

$$P = DBCW (b/W)^m (v_{pl}/W)^{1/n} \tag{1}$$

In Eq. (1) W and B are width and thickness, respectively; C and m are the geometry function parameters, with values 1.553 and 2.236, respectively, and D and n are material-dependent adjustable parameters. The variables ligament, $b = W - a$, and plastic displacement v_{pl} , have been normalized by the specimen width W . For a non-growing crack, the ligament size b is constant, and P and v become the only variables of the function, at constant crack size. Thus, by setting $b = b_o$ as the initial ligament size, Eq. (1) gives the relation between force and plastic displacement for the blunt notch specimen. It then remains to add to the plastic displacement the elastic component, which is generated with the compliance function (Donoso and Landes, 2001), at any value of P . This procedure gives the key curve KC of Figure 1 for the specimen with constant ligament size b_o . For this specimen, the material adjustable parameters $D = 283$ MPa and $n = 5.85$.

When there is stable crack extension, however, the ligament size b also becomes a variable, so that a separate relation between ligament b and v_{pl} was postulated in the form of the DZL “crack growth law” (Donoso *et al.*, 2005a; Donoso *et al.*, 2005b). Equation 2 shows the power law relation between stable crack extension Δa , and plastic displacement v_{pl} :

$$\frac{\Delta a}{W} = l_o \left[\frac{v_{pl}}{W} \right]^{l_1} \tag{2}$$

In Eq. (2), l_o is a coefficient to be determined and l_1 an exponent, which for C(T) specimens varies between 1.1 and 2.5 (Donoso and Landes, 2010). In more general terms, l_o and l_1 may also be considered adjustable parameters for the construction of a P - v curve in which there is a changing crack size in the same way as D and n are material-dependent adjustable parameters. The crack extension, Δa , may also be written in terms of the decrease in ligament size, that is, $\Delta a = b_o - b = \Delta b$. From Eq. (2), the expression for the current ligament size, b , is:

$$\frac{b}{W} = \frac{b_o}{W} - l_o \left[\frac{v_{pl}}{W} \right]^{l_1} \tag{3}$$

Substitution of Eq. (3) into the geometry term of Eq (1) gives the following expression for the CFE

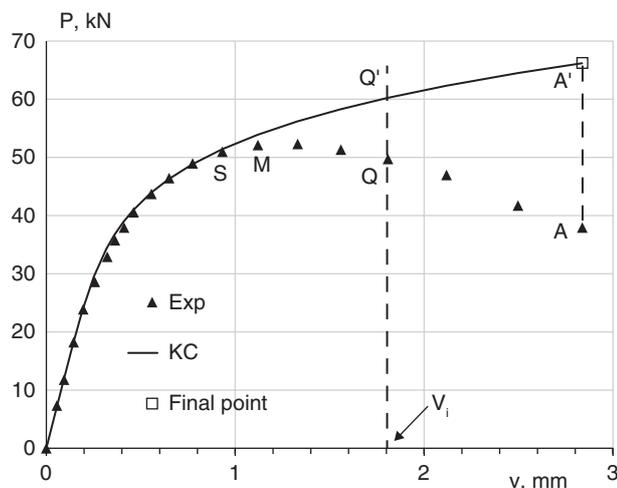


FIGURE 1. Experimental curve and key curve for A508.

as a function of plastic displacement, when there is stable crack growth:

$$P = \text{DCBM} \left[\frac{b_o}{W} - 1_o \left[\frac{v_{pl}}{W} \right]^{l_1} \right]^m \left[\frac{v_{pl}}{W} \right]^{l/n} \quad (4)$$

Thus, Eq. (1) gives the relation between force and plastic displacement for a blunt notch specimen by setting $b = b_o = \text{constant}$, whereas Eq. (4) represents the relation when there is stable crack growth, or variable ligament, b . In this latter equation, the first bracket contains the crack growth law. In Eq. (1), on the other hand, there is no crack extension, so the first bracket should include only a constant term, b_o/W .

Let us assume now that in Eq. (4), $P = P(\text{Exp})$, i.e., the C&C formats are able to reproduce the experimental curve with the appropriate parameters C , D , l_o and l_1 , as has been shown previously (for example, Donoso *et al.*, 2008) and in Eq. (1) $P = P(\text{KC})$. Then if one divides Eq. (4), with variable ligament b , by Eq. (1) with constant ligament size b_o , at constant plastic displacement, the following expression remains once all common terms have been eliminated:

$$\frac{P^{\text{Exp}}}{P^{\text{KC}}} = \left[\frac{b}{b_o} \right]^m \quad (5)$$

In Fig. 1, the key curve and the experimental curve overlap up to a point like “S”, where the two curves diverge, and the test specimen should experience initiation of crack extension following crack blunting. While the experimental curve will display a maximum in force (point “M”), the force on the key curve will continue increasing due to work hardening, and, as a requirement of the construction, should pass through point A’, which is the experimental final point A corrected by the ratio of the ligaments of the blunt-notch (b_o) and the experimental (b_f) specimens, as given by Eq. (6):

$$P_{\text{final}}^{\text{KC}} = P_{\text{final}}^{\text{Exp}} \left[\frac{b_o}{b_f} \right]^m \quad (6)$$

In Eq. (6), b_o and b_f are initial and final ligament sizes, measured on the surface of the halves of the broken test specimen, and m is the geometry exponent of the Common Format, equal to 2.236 for the C(T) specimen.

The fact that the final P - v point on the key curve (A’) has been obtained by correcting the final experimental point, A, may then be applied to other points of the test specimen, for example, point Q. In Fig. 1, at point Q, $v \sim 1.81$ mm; $P \sim 50$ kN, and the crack size measured by the unloading compliance method is 28.14 mm. The initial crack size for the A508 specimen is 26.18 mm, so that application

of Eq. (1) gives a value of ~ 61 kN for the force at Q’. This way, one can create a key curve by taking points from the experimental curve - of which all three variables: v_i , P_i , a_i are known - and generating at any point such as Q, at a given total displacement, the corresponding corrected point Q’ by means of the ligament ratios shown in Eq. (7):

$$P_i^{\text{KC}} = P_i^{\text{Exp}} \left[\frac{b_o}{b_i} \right]^m \quad (7)$$

Thus, the task at hand is to show that if at the same fixed total displacement, the elastic displacements for the experimental curve and for the blunt notch curve with $b = b_o$ are equal - or at least, similar - then the latter should play the role of a **key curve**. This means that in an experimental curve in which only the initial and final crack sizes are known, the ligament sizes b_i may be obtained by inverting Eq. (7), namely:

$$b_i = b_o \left[\frac{P_i^{\text{Exp}}}{P_i^{\text{KC}}} \right]^{1/m} \quad (8)$$

at any point with total load line displacement v_i , such as points Q (exp) and Q’(KC).

The proof proceeds as follows: the elastic components may be calculated from the compliance values corresponding to the crack (ligament) sizes at points Q and Q’. Thus, at the experimental point Q:

$$v_{el,i}^{\text{Exp}} = c_i P_i^{\text{Exp}} \quad (9)$$

And at the corrected curve point Q’:

$$v_{el,i}^{\text{KC}} = c_o P_i^{\text{KC}} \quad (10)$$

In equations (9) and (10), c_i and c_o are the elastic compliances for ligament sizes b_i (Q) and b_o (Q’), respectively. From the Concise Format (Donoso and Landes, 2001), the elastic compliance for a ligament size b_j may be expressed in the form:

$$c_j = \frac{1}{C^* E' B_e} \cdot \frac{1}{[b_j / W]^\mu} \quad (11)$$

In Eq. (11), the Concise Format parameters $C^* = 0.1315$ and $\mu = 2.283$, and E' and B_e have their usual meanings. For convenience, let $N = 1/[C^* E' B_e]$ so that the elastic components become:

$$v_{el,i}^{\text{Exp}} = \frac{N}{[b_i / W]^\mu} P_i^{\text{Exp}} \quad (12)$$

for the test curve, and

$$v_{el,i}^{KC} = \frac{N}{[b_o/W]^\mu} P_i^{KC} \quad (13)$$

for the key curve.

However, P_i^{KC} is related to P_i^{Exp} by means of Eq. (7), so that Eq. (13) may now be written as

$$v_{el,i}^{KC} = \frac{N}{[b_o/W]^\mu} P_i^{Exp} \left[\frac{b_o/W}{b_i/W} \right]^m \quad (14)$$

Multiplying numerator and denominator of the right hand side of Eq. (14) by $(b_i/W)^\mu$ and rearranging, leads to:

$$v_{el,i}^{KC} = \frac{N}{[b_i/W]^\mu} P_i^{Exp} [b_i/b_o]^{\mu-m} \quad (15)$$

By substituting Eq. (12) into Eq. (15), one gets:

$$v_{el,i}^{KC} = v_{el,i}^{Exp} \cdot \beta \quad (16)$$

Where

$$\beta = [b_i/b_o]^{\mu-m} \quad (17)$$

From the Concise Format (Donoso and Landes, 2001), $\mu = 2.283$, whereas from the Common Format (Donoso and Landes, 1994), $m = 2.236$. These values differ only by 2.1%, so that if one ignores this difference (i.e., $\mu = m$), then $\beta = 1.0$ and the elastic components become equal. On the other hand, if one takes this difference as significant, at the last point on the experimental curve of Fig. 1, where $a_f = 31.62$ mm, so that $b_f = 17.69$ mm, and $\mu - m = 0.047$, then $\beta = 0.988$. This gives a difference in the elastic components at the final point of the test of 1.2%. At points Q and Q', however, $\beta = 0.995$, and one can assume with all confidence that the elastic components are equal.

Thus, at any point Q along the experimental curve with total displacement v_i , there will be a mirror point Q' on the key curve such that the elastic components are, for all intents and purposes, equal. The implication of this assertion is that the plastic displacement, calculated as total displacement minus the elastic component, will also be equal. This was the assumption that led to the construction of the key curve, Fig. 1, passing through point A', and was shown in an earlier work by Donoso *et al.* (2009).

The significance of this finding is far from trivial. Let us assume now that an experiment has been performed in which there is crack extension, and a large number of points along the curve are known as P - v pairs, with no simultaneous measurement of crack sizes as the test proceeds: only the initial and the final crack sizes have been measured from the broken specimen. This, as explained earlier, may be the result of a varied number of circumstances: high temperature, hostile environment, dynamic testing, missing data, or simply the lack of an adequate (and expensive) infrastructure to measure crack extension concurrently with force and displacement, in order to construct a J - R curve.

In order to produce crack (ligament) sizes at any P - v pair beyond a point such as "S" in Fig. 1, then Eq. (8) may be instrumental in obtaining the missing crack size data that will ultimately allow the construction of a valid J - Δa curve for that specimen. ASTM E1820 Standard has a number of requirements that need to be fulfilled in order to construct a J - R curve that delivers an initiation value, J_{Ic} , among which are notable the number and spacing of Δa values. Therefore, the key curve, obtained by resorting to the Common and Concise Formats, plays the role of a source curve from which "target" curves with missing crack extension values may be constructed. This is the core subject of this paper.

3. RESULTS

3.1. Application of the model to the A508 C(T) specimen

The procedure was used on the A508 specimen, and the results are shown in Figs. 2 and 3. Computed crack sizes $a(KC)$ were obtained by calculating the ligament size on the mirror point Q on the target (experimental) curve, from points like Q' on the key curve of Fig. 1. The computed crack sizes $a(KC)$ are then compared to the experimentally measured crack sizes $a(Exp)$, and shown in Fig. 2. Initial and final crack sizes are included for comparison purposes, and joined by a dashed line.

The result is quite good, considering that the key curve is a continuous, "perfect" curve, and the experimental curve may show some ripples, this being the main cause for the difference in crack sizes at any point on the target curve, as may be inferred from Fig. 2. An important proof that the methodology works appropriately, is that the crack sizes for the target specimen, calculated from the key curve and used in Eq. (4), allow reconstructing the specimen experimental curve. This is shown in Fig. 3 by the open circles curve labeled C&C; this C&C curve has been constructed with the following parameters: $D = 283$ MPa; $n = 5.85$; $l_o = 56$, and $l_l = 2.0$. In this figure, the key curve has been drawn as a continuous curve up to the final point.

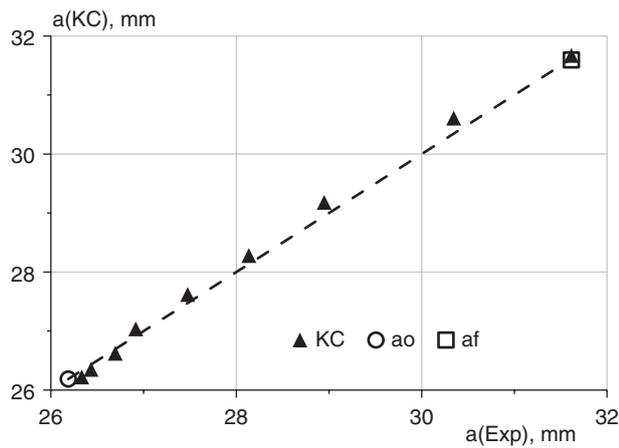


FIGURE 2. The resulting crack sizes from application of Eq. (8).

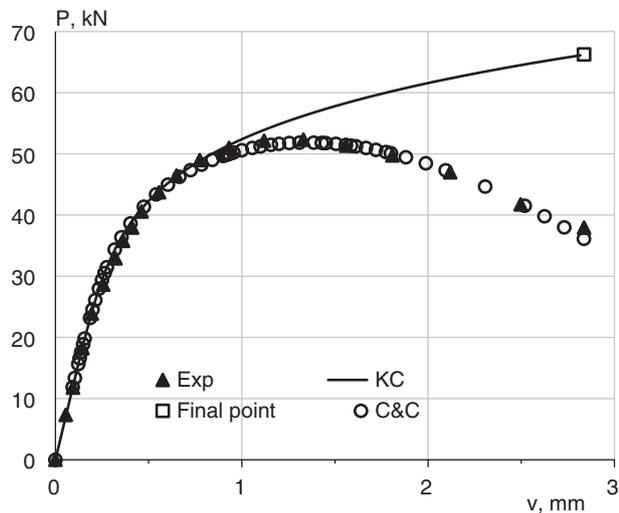


FIGURE 3. Experimental, key and C&C curves for A508.

The next step is the construction of the $J-R$ curve for the specimen, shown in Fig. 4. The construction line and the 0.15, 0.20, 0.50 and 1.50 mm exclusion lines have been drawn with a slope of $2\sigma_Y = 990$ MPa, given that the yield and the ultimate tensile stress for this material are 385 MPa and 605 MPa, respectively. The values of J_{limit} and Δa_{limit} are $1625 \text{ kJ}\cdot\text{mm}^{-2}$ and 1.8 mm respectively. The number of experimental $P-v$ points is insufficient to provide a valid $J-R$ curve. The use of the key curve, on the other hand, allows for the inclusion of a great number of $J-\Delta a$ values with which to construct a curve that follows the power law expression $J = C_1(\Delta a)^{C_2}$ defined in E1820. For the crack extension limits given in the standard, such curve is characterized by the parameters: $C_1 = 284.9$; $C_2 = 0.456$ and $R^2 = 0.9998$.

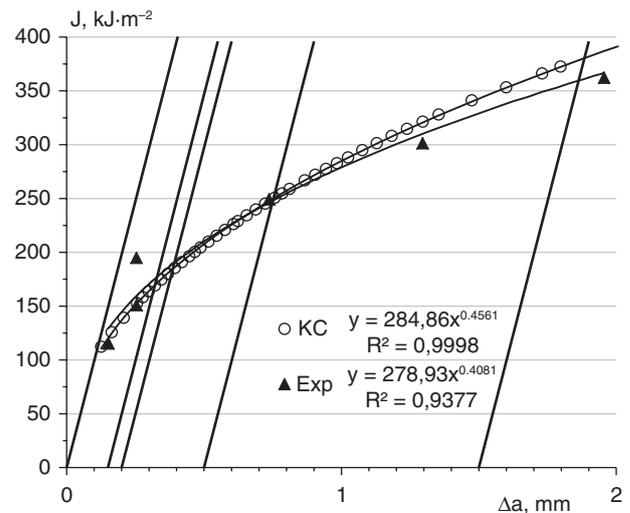


FIGURE 4. The $J-R$ curves for the experimental and key-curve derived A508 data.

The value of J_Q is $185 \text{ kJ}\cdot\text{mm}^{-2}$, and the quantity defined in E1820 as $10J_Q/\sigma_Y$ has a value of 4.8 mm, much smaller than B and b_o . Therefore, $J_Q = J_{Ic}$, and $K_{Jc} = 192 \text{ MPa}\sqrt{\text{m}}$. For this calculation, the modulus E was taken as 200 GPa.

This result is quite encouraging, and the key curve procedure to generate crack sizes may be summarized as follows: run fracture tests on two identical C(T) specimens, one with a sharp crack of a given size, and another with a blunt notch of the same size. Application of Eq. (8) at various total displacements will produce the value the crack will have as long as crack extension and plastic deformation takes place in the pre-cracked specimen. At the end of the test, the final crack size of the pre-cracked specimen may be used to obtain the corrected force at point A' . As explained earlier and shown in Fig. 1, point A' should be on the blunt notch $P-v$ curve, or Key Curve.

3.2. The Key Curve for a TWIP specimen

One interesting case of a $P-v$ curve with limited number of crack sizes available corresponds to a TWIP (Twinning Induced Plasticity) steel C(T) specimen tested at room temperature (De Barbieri, 2014). The material is austenitic steel with base composition 0.6% C and 22% Mn. Tensile testing of this steel shows a yield stress of 430 MPa, an ultimate tensile stress (UTS) of 905 MPa, and a large engineering strain of the order or 60% at the UTS. Therefore, this TWIP specimen shows a very high work hardening rate, due to a combination of twinning (TW), dynamic strain aging (DSA), and dislocation glide (DG).

The TWIP C(T) specimen has dimensions $W = 49.48$ mm, $B = 19.15$ mm, and initial and final

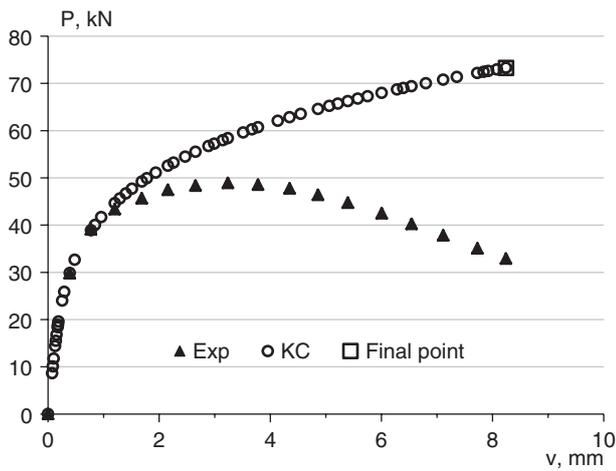


FIGURE 5. Experimental curve, key curve and final point for the TWIP specimen.

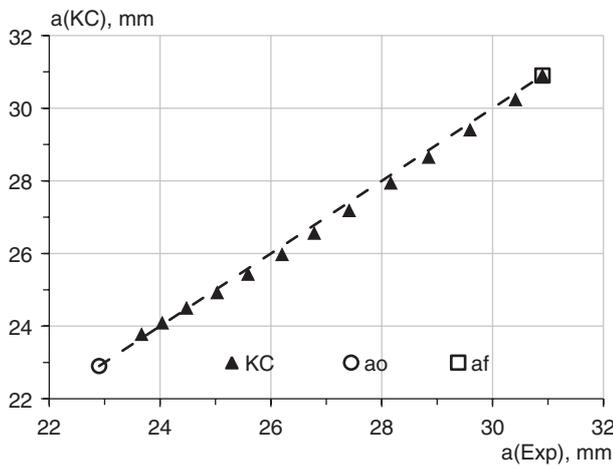


FIGURE 6. Key curve generated crack sizes vs experimental values for the TWIP specimen.

crack sizes $a_o = 23.0$ mm and $a_f = 30.9$ mm respectively. Crack sizes were measured by the unloading-reloading compliance method at 13 points of a total of 16 points on the $P-v$ curve. The experimental curve, the key curve (material parameters $D = 303$ MPa and $n = 4.5$) and the final corrected point are shown in Fig. 5, which contains all the information needed to obtain crack sizes for the TWIP specimen using the present Key Curve methodology.

The Key Curve for the TWIP specimen was generated in such a way that the experimental points with measured crack sizes could have a matching point on the Key Curve at the same total displacement. Thus, the Key Curve generated crack sizes were compared to the experimentally measured crack sizes for those same points. Initial and final crack sizes are also included; the result is shown in

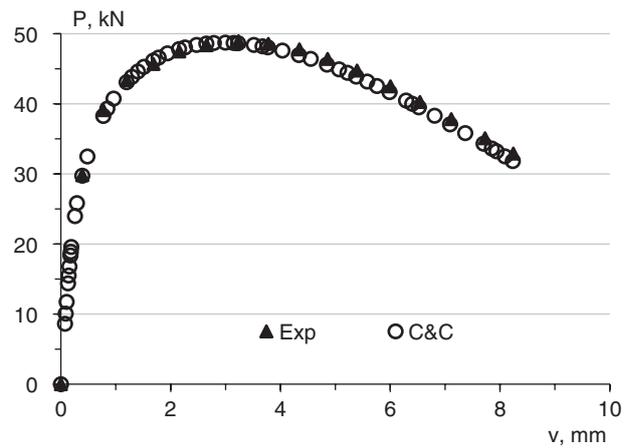


FIGURE 7. C&C and experimental curves for the TWIP specimen.

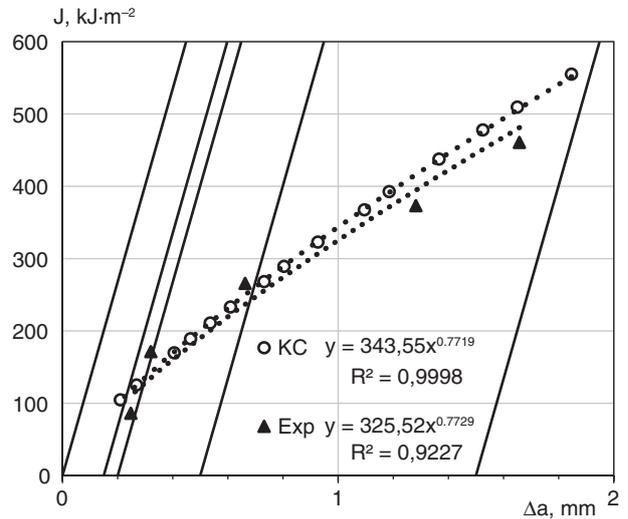


FIGURE 8. Regression analysis for the C&C and the experimental $J-R$ curves.

Fig. 6. The crack sizes obtained from this procedure were then used to generate the crack growth law of Eq. (2): the parameters obtained from a power law regression for the TWIP specimen are $l_o = 2.10$ and $l_l = 1.35$, with $R^2 = 0.998$.

The $P-v$ curve with crack growth for the TWIP specimen was constructed with as many points as needed in order to generate a $J-R$ curve with sufficient Δa values to be valid by E1820. This is shown in Fig. 7, where the C&C curve, incorporating crack extension data by means of the crack growth law, practically matches the experimental curve. Let us remember that the actual curve has a total of only 16 points; the C&C curve shown in Fig. 8 has 55 data points, i.e., more than three times the number of points as the original curve.

The next step is the *J-R* curve construction. The experimental and C&C *J-R* curves for the TWIP specimen, between the 0.15 and 1.5 mm exclusion lines, are shown in Fig. 8. The construction line and the 0.15 mm, 0.2 mm, 0.5 mm and 1.5 mm lines have been drawn with a slope $2\sigma_Y = 1335$ MPa, which corresponds to the sum of yield stress (430 MPa) and UTS (905 MPa) of the TWIP specimen. The value of Δa_{min} and Δa_{limit} defined in E1820 are 0.22 mm and 1.90 mm respectively, whereas $J_{limit} = 2350$ [kJ·m⁻²]. For clarity purposes, the maximum value of *J* in Fig. 8 has been set to 600 [kJ·m⁻²].

It should be noted that the number of the experimental points within the *J* and Δa limits set by E1820 - five *J-Δa* points in this case, from De Barbieri's thesis - barely fulfill the requirements of the Standard. The key curve procedure described here, on the other hand, allows for the inclusion of as many points as are needed to produce a valid J_Q as per E1820.

The regression analysis for the experimental and the C&C *J-Δa* data points are also shown in Fig. 8. The regression line of the form $J = C_1(\Delta a)^{C_2}$ obtained with the experimental points has $C_1 = 325.5$, $C_2 = 0.773$ and $R^2 = 0.9227$. The line for the C&C points has $C_1 = 343.6$, $C_2 = 0.772$ and $R^2 = 0.9998$. With the latter, a J_Q of 135 [kJ·m⁻²] is obtained at $\Delta a_Q \approx 0.30$ mm. The size validity checks for initial ligament and thickness give a value of 3.2 mm for the quantity $10J_Q/\sigma_Y$, a value which is largely surpassed by *B* and b_o . On the other hand, the slope of the curve at Δa_Q has a value of 350 MPa, compared to the value of $\sigma_Y = 668$ MPa.

Having fulfilled the E1820 validity criteria that deal with number and spacing of data points; the quality of the correlation concerning C_2 and R^2 ; the size validity concerning *B* and b_o , and the value of the slope of the regression line at Δa_Q , a value of J_{Ic} of 127 [kJ·m⁻²] should lead to a value of K_{JIc} of 164 MPa√m, using a value of *E* of 200 GPa.

4. DISCUSSION

The construction and use of a Key Curve in the evaluation of crack size data has been studied by Joyce (1983); Andrews (1985); Joyce *et al.* (2001); Candra *et al.* (2002) and Emrich *et al.* (2007). Candra *et al.* (2002) wrote a thorough summary on the derivation of relations between key and target curves. Regarding the construction of a Key Curve for C(T) specimens, these authors assumed that the elastic components of the displacement in both key and target curves, are equal at any given total displacement, from which the forces on key and target curves must be inversely proportional to the corresponding elastic compliances. Both of these most relevant aspects in the construction of a Key Curve were demonstrated and confirmed in this paper with the use of the Common and Concise Formats.

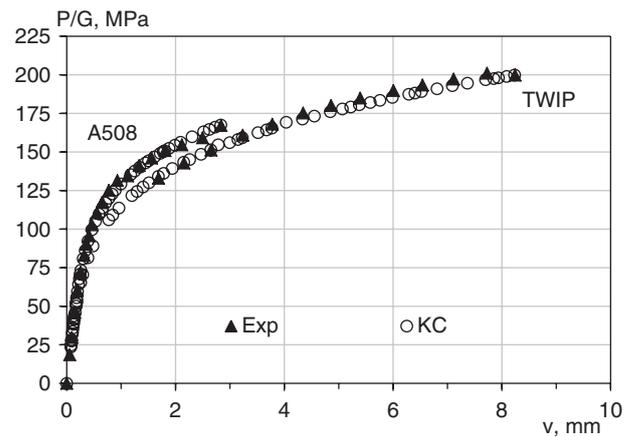


FIGURE 9. Normalized curves (force axis) for the A508 and TWIP specimens.

Figure 9 shows two sets of curves for the TWIP and the A508 specimens, in the manner of force normalized by the C&C geometry function *G*, vs total displacement *v*. For each specimen, the experimental force has been normalized by the function $G = CBW(b_i/W)^m$, which changes as the ligament b_i decreases due to crack extension. The result is shown by the black triangles in Fig. 9 for both specimens. Overlapping the two normalized experimental curves, the corresponding Key Curves, for which there is no crack extension, have been normalized by the constant term $G_o = CBW(b_o/W)^m$, and are shown by the open circles. From the results shown in Fig. 9, it becomes apparent that for each specimen, the two types of curves are coincident, a fact that may be expressed as:

$$\frac{P_i^{Exp}}{G} = \frac{P_i^{KC}}{G_o} \tag{18}$$

The equality of Eq. (18) implies then that for each specimen the two normalized curves are one and the same. Thus, the two sets of curves of Fig. 9 illustrate that, at any given total displacement, the experimental force normalized by the variable quantity *G* is equal to the specific Key Curve normalized by the constant term G_o . This statement may be written as:

$$\frac{P_i^{Exp}}{CBW[b_i/W]^m} = \frac{P_i^{KC}}{CBW[b_o/W]^m} \tag{19}$$

After removing all common terms, Eq. (19) will deliver the same result as Eq. (7). Thus, Eq. (7) and Eq. (8), which are the fundamental relations for the construction of force vs crack size data, should be rewritten as:

$$P_i^{KC} = P_i^{Exp} \left[\frac{b_o}{b_i} \right]^m \tag{20}$$

And

$$b_i = b_o \left[\frac{P_i^{Exp}}{P_i^{KC}} \right]^{1/m} \quad (21)$$

Summarizing, the steps required to determine crack sizes with the aid of the C&C derived Key Curve, are:

- a. Conduct a fracture test on a pre-cracked 1T-C(T) specimen with the usual dimensions suggested by E1820-15 (2015), and initial crack size $a_o/W \sim 0.5 - 0.7$. Measure the values of initial and final crack sizes on the broken halves of the pre-cracked specimen, thus determining average values of a_o and a_f .
- b. Run a similar test on a blunt-notch 1T-C(T) specimen with the same dimensions and initial crack size equal to the average a_o determined above. Construct the P - v curves in the usual manner for both the pre-cracked and the blunt-notch specimens.
- c. From the force value of the final point A of the pre-cracked specimen test (see Fig. 1), evaluate the corrected force value for the final point A' , by using Eq. (6). The P - v curve of the blunt-notch specimen should go through this final corrected point, becoming the “Key Curve” for that pre-cracked specimen.
- d. If it is not possible to run a blunt-notch test, construct the Key Curve with the aid of Eq. (1), with $b = b_o$, the average initial ligament size of the specimen tested. At this point, the elastic displacement v_{el} should be added to the plastic component, v_{pl} , to give total displacement v . Care should be taken with the values assigned to the C&C adjustable parameters D and n , so that the key curve should pass through point A' defined above.
- e. At any given displacement larger than that for which the experimental and the key curve diverge (like point “S” of Fig. 1), proceed to evaluate the ligament sizes for the experimental curve with Eq. (21), at constant total displacement.
- f. The calculations of step e above should yield a sizable number of data points for the experimental curve of the type $\{P, v, b\}$, where v is total displacement. The value of the crack size a at any point is $a = W - b$.
- g. Use the elastic compliance to obtain the elastic displacement for the experimental curve at any P - v - a point. The difference between total and elastic displacement will give the plastic displacement, required to formulate the crack growth law, Eq. (2) and obtain the adjustable parameters l_o and l_f .
- h. Use Eq. (4) to construct a P - v curve in which now there is a changing ligament size

(crack extension). As stated in step d, the elastic displacement should be added to give total displacement. This P - v curve should match the experimental curve point by point. This aspect validates the crack size calculations with the C&C model.

- i. Now a J - R curve with sufficient Δa values may be constructed. What follows next is to generate the values of C_1 and C_2 , and to validate J_Q by the requirements of ASTM Standard E1820-15 (2015).

5. CONCLUSIONS

- A Common and Concise Formats approach to a Key Curve construction for generating crack sizes in C(T) specimens, when there are not enough data to validate a J - R curve, has been presented. The key curve is a *corrected force* type of curve, in the sense that it is derived from the experimental curve, for which the whole P - v data are known, but only initial and final crack sizes are determined at the end of the test. By using the appropriate values of the adjustable material parameters D and n , the Key Curve, for which there is no crack extension, may be constructed in such a way that it should pass through the final corrected point of the experimental curve.
- The construction of the Key Curve is based on the Common and Concise (C&C) Formats, and therefore, it has analytical support. On this basis, the elastic compliances of both the Key Curve and the experimental curve may be matched not only at the end of the actual test (final points A and A' of Fig. 1), but at any point along the Key Curve. Thus, it has been shown that at a given value of total displacement, the elastic components of both the key and the experimental curve are, for all purposes, equal. This implies that the plastic displacements of Key Curve and experimental curve are also equal.
- It has also been shown that at any given total displacement, the experimental force normalized by the variable quantity G is equal to the specific Key Curve normalized by the constant term G_o . The meaning of this equality, given by Eq. (19), is that the Key Curve, normalized by a constant term (G_o) and the target (experimental) curve, normalized by the function $G(b)$, are one and the same. Therefore, a normalization process through the C&C formats should lead to the Key Curve for that material and geometry.
- The Key Curve method presented here has some advantages over the previous methodologies developed by Donoso and coworkers, i.e., the Crack Growth Law (Donoso *et al.*, 2005a; Donoso *et al.*, 2005b) and the Intercept Method

(Donoso *et al.*, 2008; Donoso *et al.*, 2009). All three methods - Crack Growth Law, Intercept Method, and Key Curve Method - are based upon the Common and Concise Formats, and therefore have robust analytical support. Although under given circumstances all three may be used as complementary tools, the Key Curve concept presented here is the closest to a normalization procedure, such as that suggested in Annex A15 of Standard ASTM E1820-15 (2015). Within this context, the C&C based Key Curve method was applied to two cases of fracture testing of C(T) specimens in which a limited number of crack sizes were available as a function of force and displacement (A508 and TWIP).

- As explained earlier, these methodologies — the Crack Growth Law, the Intercept Method and the present C&C based Key Curve method — have been used up to now with data solely from C(T) steel specimens, with $W = 50$ mm. Work on other geometries, materials and sizes should be encouraged in order to improve one or more of these methodologies in order to generate crack sizes where they are missing, by an adequate use of blunt-notch tests, or by using the C&C Formats to construct Key Curves.

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